

Introduction

In this paper, we are to discover *structural inconsistencies*, i.e., nodes that connect to a number of diverse influential communities in the network (Fig. 1).

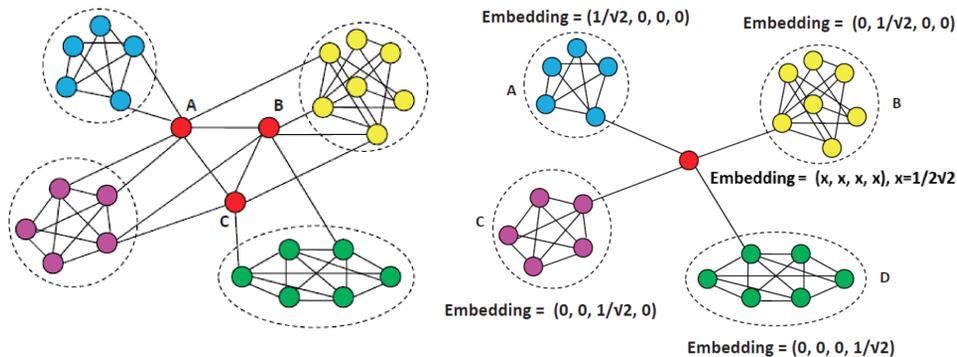


Figure 1: anomalous (red) nodes

Figure 2: nodes in embedding

Relationship with structural hole brokers. In Burt's structural hole theory, an individual (broker) who acts as a mediator between two or more groups of people (e.g., A-C in Fig. 1) would gain important social capital such as novel ideas [1]. In this sense, structural inconsistencies also provide a formal definition for structural hole brokers.

Impacts of anomalies. Since the anomalous nodes connect to diverse regions in the network, the incident links violates the notion of *homophily* [2], which assumes that linked nodes have similar properties. Because of this inconsistency in the link structures, the presence of such anomalies may:

- have a substantial impact on network structure, e.g., nodes from four groups tend to form one large group in Fig. 1;
- prevent effective application of many network mining algorithms, e.g., hard to achieve meaningful clusters.

Graph Embedding

To detect structural inconsistencies, we first use graph embedding to associate each node with a multidimensional position. In the embedding model, each dimension corresponds to a clustered region in the network.

Given an undirected graph $G=(V,E)$, associate each node i with a d -dimensional vector X_i , which represents the correlation between node i and the d communities (Fig. 2). The goal in this embedding is to preserve linkage structure of the network. Finally, the embedding is determined by *minimizing the objective function* O :

$$O = \sum_{(i,j) \in E} \|X_i - X_j\|^2 + \alpha \cdot \sum_{(i,j) \notin E} (1 - \|X_i - X_j\|)^2,$$

where α is a balancing factor, which regulates the importance of the two components in O .

A Quantitative Measure of Anomaly

After deriving the embedding, anomalous nodes are determined using the embedding together with a quantitative measure.

We first define $NB(i)$ to evaluate the correlation of node i with the d communities (instead of using X_i alone):

$$NB(i) = (y_i^1, \dots, y_i^d) = \sum_{(i,j) \in E} (1 - \|X_i - X_j\|) \cdot X_j$$

Given $NB(i)$, we introduce the $AScore$ measure to indicate the anomalousness level of node i :

$$AScore(i) = \sum_{k=1}^d \frac{y_i^k}{y_i^*}, y_i^* = \max \{y_i^1, \dots, y_i^d\}$$

Finally, node i is detected as an anomaly if $AScore(i) > thre$.

In fact, $AScore$ measure is also a quantitative measure for detecting structural hole brokers.

Algorithm Optimizations

By now, the $O(n^2)$ terms in O make our approach hard to be applied to large networks. Hence, we further use the sampling and graph partitioning, and propose a novel dimension reduction technique, to make our approach more scalable and effective for large networks.

Sampling. It is very inefficient to express O as a sum of $O(n^2)$ terms. An observation here is that α is typically picked close to 0 and it is possible to approximately represent O by sampling a subset E_s of size $|E|$ for the second component:

$$O \approx \sum_{(i,j) \in E} \|X_i - X_j\|^2 + \sum_{(i,j) \in E_s} (1 - \|X_i - X_j\|)^2, E_s \subset \{(i,j) | (i,j) \notin E\}$$

Graph partitioning based initialization. We use a gradient descent method to optimize O , which is critically dependent on a good initialization. Thus, we incorporate graph partitioning for initialization such that densely connected nodes are initialized with similar embedding values (Fig. 2).

Dimension reduction. The number d can be large in practice, while anomalies typically connect to a limited number of communities. This motivate us to only maintain $(k+\beta)$ -dimensions for embedding of each node. Numbers k and β could be much smaller, e.g., 10 and 2.

Experimental Results

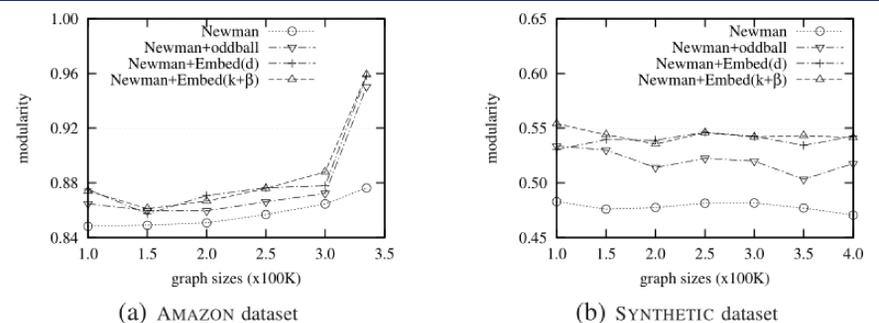


Figure 3: improvement on effectiveness of community detection (modularity)

The removal of detected anomalies helps improve the effectiveness of community detection.

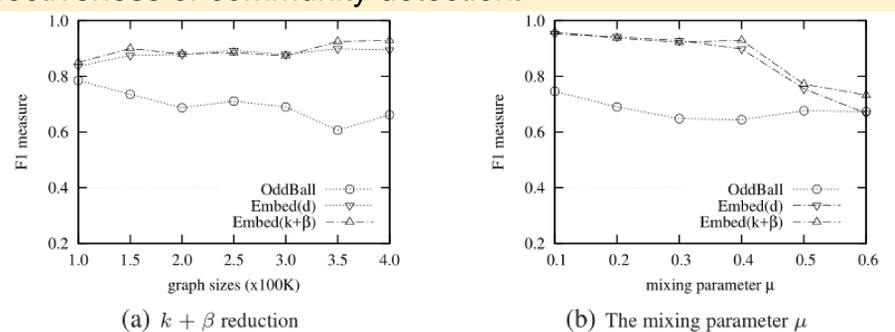


Figure 4: quality (F_1 measure) comparison on SYNTHETIC dataset

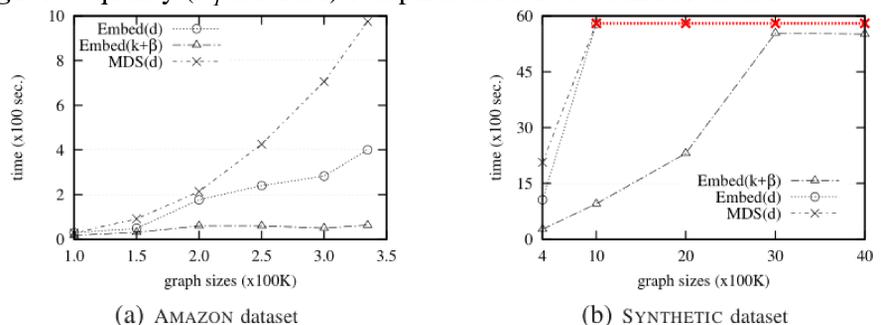


Figure 5: efficiency comparison w.r.t the graph sizes

Our embedding approach to anomaly detection is both effective and efficient. Moreover, the $(k+\beta)$ reduction technique reduces space cost and improves efficiency in the same time, and slightly improves effectiveness.

References

- [1] R. S. Burt. Structural holes and good ideas. *American Journal of Sociology*, 110(2): 349-399, 2004.
- [2] M. McPherson, L. Smith-lovin, and J. Cook. Birds of a feather: Homophily in social networks. *Annual review of sociology*, Vol. 27: 415-444, 2001.