Flexible Aggregate Nearest Neighbor Queries in Road Networks

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***Abstract*—Aggregate nearest neighbor (ANN) query has been studied in both the Euclidean space and road networks. The flexible aggregate nearest neighbor (FANN) problem further generalizes ANN by introducing an extra flexibility. Given a set of data points P, a set of query points Q, and a user-defined flexibility parameter ϕ that ranges in $(0, 1]$, an FANN query returns the best candidate from P, which minimizes the aggregate (usually *max* or *sum*) distance to any $\phi|Q|$ objects in Q. In this paper, we focus on the problem in road networks (denoted as FANN$_\mathcal{R}$), and present a series of universal (i.e., suitable for both max and sum) algorithms to answer FANN$_\mathcal{R}$ queries in road networks, including a Dijkstra-based algorithm enumerating P, a queue-based approach that processes data points from-near-to-far, and a framework that combines *Incremental Euclidean Restriction* (IER) and kNN. We also propose a specific exact solution to *max*-FANN$_\mathcal{R}$ and a specific approximate solution to *sum*-FANN$_\mathcal{R}$ which can return a near-optimal result with a guaranteed constant-factor approximation. These specific algorithms are easy to implement and can achieve excellent performance in some scenarios. Besides, we further extend the FANN$_\mathcal{R}$ to k-FANN$_\mathcal{R}$, and successfully adapt most of the proposed algorithms to answer k-FANN$_\mathcal{R}$ queries. We conduct a comprehensive experimental evaluation for the proposed algorithms on real road networks to demonstrate their superior efficiency and high quality.**

I. Introduction

The aggregate nearest neighbor (ANN) query [1]–[7] is a classic problem that has a large number of applications (e.g., location-based services) in spatial databases. Given a group of query points Q, ANN finds out a point in a set of data points P, which has the smallest aggregate distance to *all* points in Q. The aggregate function is usually either *max* or *sum*. The ANN problem has been studied in both the Euclidean space [1]–[3] and road networks [4]–[7].

In many cases, it is more desirable to take a fraction of query points Q into account. Consider the example of Fig. 1, where a set of data points $P = \{p_1, p_2, \ldots, p_8, p_9\}$ (colored in black), and a set of query points $Q = \{q_1, q_2, q_3, q_4\}$ (colored in red). Note that p_4 and q_3, p_5 and q_4 are located at the same node, respectively. Some points are located at edges. For example, q_1 lies on (p_2, p_3), and q_2 lies on (p_3, p_6). In an online war strategy game, Q is a set of small military camps, and there is a fixed number of soldiers in each camp. Players should choose the best place from a set of candidate locations (i.e., P) to build a logistics center. If the supplies are abundant enough, the possible choice can be answered by an ANN query. The result of this *max*-ANN query is p_2 with the aggregate distance of 16 (from p_2 to q_4) and the result of this *sum*-ANN query is also p_2 with the aggregate distance of 52 (i.e., $10+14+12+16$). The paths from p_2 to Q have been colored in green, and we add a "virtual" edge from p_2 to q_2 for better visualization. We can intuitively understand the result because p_2 is the geographical "center" of Q. However, if this logistics center can only afford to support 50% camps due to the limited supplies in the game settings, the result is different. In this case, for the sake of players' victory, the possible strategy is to find the best place to build a logistics center which is able to supply 50% of these camps while minimizing the aggregate traveling cost. More precisely, a more general query is to allow users to specify a flexibility parameter $\phi \in (0, 1]$, and the goal is to retrieve the best point from P that is the closest to *any* $\phi|Q|$ points in Q. We denote this query as the flexible aggregate nearest neighbor in road networks (FANN$_\mathcal{R}$). When ϕ is 50%, it is obvious that the result of this *max*-FANN$_\mathcal{R}$ query is p_3 with the aggregate distance of 2 (from p_3 to either q_1 or q_2) and the result of this *sum*-FANN$_\mathcal{R}$ query is also p_3 with the aggregate distance of 4 (i.e., $2+2$). The paths from p_3 to these $\phi|Q|$ points have been colored in blue.

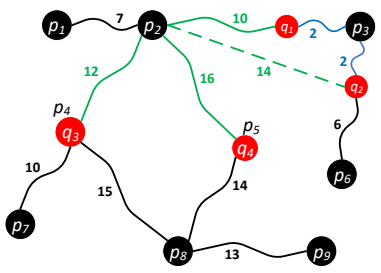

Fig. 1. Example of an FANN$_\mathcal{R}$ query in road networks

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As illustrated above, FANN$_\mathcal{R}$ can be used in online games' site-selection in road networks. To emphasize its importance, we take a real world scenario as an example. *Choosing a location to hold an election meeting:* Suppose that the meeting is legitimate as long as at least half of members are present. To cut down the traveling expense, we can find a place which minimizes the *flexible* aggregate (*sum*) distance to members.

To the best of our knowledge, there is no any research on FANN$_\mathcal{R}$ query problems in road networks. FANN queries [8], [9] are first studied in the Euclidean space. Compared with the Euclidean space, many operations in road networks are much more expensive. For example, determining the distance between any two points in the Euclidean space can be solved in $O(1)$ time. However, the calculation process in road networks is much more complex, and the computation cost depends on the implementation of the shortest path distance algorithm. Although it is possible to get it in $O(1)$ using pre-computation in road networks, this overhead (with respect to time and storage) of pre-processing is unbearably high. In order to further accelerate the computing in road networks, the topology nature of the road networks should be fully utilized in order to prune the unnecessary candidate points as many as possible. Some geometrical properties in the Euclidean space will not hold in road networks, and hence it is necessary to redesign the algorithms in the context of road networks.

The most relevant study is the ANN query in road networks [4], [6], [7], but our study of FANN$_\mathcal{R}$ is not a trivial extension or adaption based on ANN query for two main reasons. Firstly, the *IER* algorithm in [4] can reach the best performance, which uses R-tree to index the data objects, but it is not efficient when only partial points in Q are considered. To be specific, because *any* $\phi\,|Q|$ objects in Q may be the potential targets, the number of all possible answers can achieve the scale of $\binom{|Q|}{\phi|Q|}$. Secondly, our study of FANN$_\mathcal{R}$ is comprehensive and it could be applied in different scenarios (e.g., having an index structure or not, returning an exact answer or not). Obviously, we can regard ANN query as a special case of FANN$_\mathcal{R}$ query when $\phi = 1$. Another relevant but different type of query is the *optimal meeting point* (OMP) query [5] in road networks, but the set P in OMP query is not determined in advance. However, the uncertainty of set P does not really increase the complexity. As proved in [5], [10], given an OMP query with a set of query points Q on a road network $G = (V, E)$, $V \cup Q$ contains an OMP. In other words, we can determine the set P in an implicit way. Hence, we can also regard the OMP query as a special case of the FANN$_\mathcal{R}$ query.

In this paper, we first focus on the universal (i.e., applicable to both *max* and *sum*) methods for the FANN$_\mathcal{R}$ query problem. We first design a Dijkstra-based algorithm enumerating P. Secondly, we successfully modify the *List* algorithm [8], [9] in the context of road networks, which processes data points from-near-to-far. Thirdly, we design a general algorithm framework by combining *Incremental Euclidean Restriction* (IER) and kNN. Note that the IER here refers to general *Incremental Euclidean Restriction*, rather than the specific algorithm for ANN in [4]. To boost the efficiency, these universal algorithms often require complex index structures which usually are infeasible for large dynamic road networks with respect to time and space. Motivated by this, we sacrifice some generalities and investigate a specific algorithm for *max*-FANN$_\mathcal{R}$ which can give an exact answer. Sometimes, it is also desirable to retrieve a near-optimal answer as fast as possible. Therefore, we further design an efficient specific 3-approximation algorithm for *sum*-FANN$_\mathcal{R}$. We also prove that the approximation ratio can even reach 2 when Q is a subset of P. These two specific methods have little dependence on index structure over road networks, and hence they can still give acceptable answers especially when road networks change frequently (or we cannot build an index over the whole road network easily). Besides, these specific algorithms are easy to implement. Finally, we further extend FANN$_\mathcal{R}$ to k-FANN$_\mathcal{R}$, and adapt most of the proposed algorithms successfully to answer k-FANN$_\mathcal{R}$ queries in road networks.

The main contributions of our work can be summarized as follows:

- We firstly introduce an extra flexibility parameter to the classic ANN problem in road networks, and propose a series of methods to answer FANN$_\mathcal{R}$ queries in road networks.
- We design a Dijkstra-based algorithm to answer FANN$_\mathcal{R}$ queries, which is much better than adopting ANN as an independent module directly. Some researchers [8], [9] have tried the *List* to answer FANN queries in the Euclidean space. We modify this algorithm, by taking the features of road networks into account to solve FANN$_\mathcal{R}$ queries, and denote it as *R-List* (road networks' *List*). We also combine IER with kNN, and then develop a family of algorithms based on the general algorithm framework for FANN$_\mathcal{R}$ queries.
- We propose a specific *Exact-max* (exact *max*-FANN$_\mathcal{R}$) algorithm for *max*-FANN$_\mathcal{R}$ and *APX-sum* (approximate *sum*-FANN$_\mathcal{R}$) algorithm for *sum*-FANN$_\mathcal{R}$ to get the exact and 3-approximation answer, respectively. We also prove that the approximation ratio of *APX-sum* can even reach 2 if Q is the subset of P. These specific algorithms are easy to implement and can achieve excellent performance in some scenarios.
- We further extend FANN$_\mathcal{R}$ to k-FANN$_\mathcal{R}$, and formulate it formally in road networks. We also successfully adapt most of the proposed algorithms to answer k-FANN$_\mathcal{R}$ queries.

The rest of this paper is organized as follows: Section II defines the problem, discusses some related works, and shows an outline of the proposed approaches. Section III presents universal methods, including a Dijkstra-based algorithm, a queue-based method and a general algorithm framework. Section IV presents specific methods to answer *sum*-FANN$_\mathcal{R}$ and *max*-FANN$_\mathcal{R}$ queries respectively. Section V discusses k-FANN$_\mathcal{R}$, which is an extension of FANN$_\mathcal{R}$. We present the experimental results in Section VI and make a conclusion in Section VII.

II. PRELIMINARIES

We first present the road network definitions that we follow throughout the paper and formulate the FANN$_{\mathcal{R}}$ problem formally. Then, we review some related works. Finally, we present the outline of proposed methods in this paper.

A. Problem Formulation

A road network can be represented as an undirected weighted graph, $G = (V, E, W)$, where V is the set of nodes, E is the set of edges, and W is a weight function $E \to \mathbb{R}^+$, which maps an edge to a positive real number.

We use Q to denote the set of query objects, and use P to denote the set of data objects, and use ϕ to denote the flexibility parameter, and ϕ varies in the range of $(0, 1]$. For simplicity, we assume that query (or data) objects are at vertices, i.e., $P \subset V$, $Q \subset V$. If the query (or data) object is on an edge, we can use the two vertices on the edge to do FANN$_{\mathcal{R}}$ search and merge the answer sets of the two vertices to generate the final result. If the query (or data) object is outside the whole network, we can find the closest point in the network and use it to do FANN$_{\mathcal{R}}$ search. The similar assumption is also adopted in [11]. In the following, "node", "point", "object", and "vertex" are interchangeably used if the context is clear.

Let δ be the distance function on G, and the network distance $\delta(v_i, v_j)$ between node v_i and v_j is defined as the minimum sum of weights of any path between them. We use $\delta^{\epsilon}(v_i, v_j)$ to denote the Euclidean distance between node v_i and v_j. Let g be an aggregate function, which can be defined on a single node p and a dataset $S \subset V$, and it is either *sum* or *max* in this paper:

$$g(p, S) = g\{\delta(p, v_1), \delta(p, v_2), ..., \delta(p, v_k)\}$$

where $k = |S|$, $v_i \in S$, for $i = 1, 2, \ldots, k$.

Then we can define the *flexible aggregate function* g_ϕ, which is the most critical operation for the FANN$_{\mathcal{R}}$ query problem in road networks.

Definition 1 (Flexible Aggregate Function)**.** The flexible aggregate function g_ϕ, is a function that takes a node $p \in P$ and the set Q as its input, and returns a pair (d^p, Q^p_ϕ) as the result, i.e., $(d^p, Q^p_\phi) = g_\phi(p, Q)$, which satisfies:

$$\begin{cases} Q^p_\phi = \underset{Q_\phi \subset Q, |Q_\phi| = \phi|Q|}{\operatorname{argmin}} g(p, Q_\phi), \\ d^p = g(p, Q^p_\phi), \end{cases}$$

where Q_ϕ is the subset of Q with $|Q_\phi| = \phi|Q|$. Given a p, we denote the *optimal flexible subset* of Q as Q^p_ϕ, and denote its *flexible aggregate distance* to Q as d^p.

It is worth noting that we usually use $g_\phi(p, Q)$ to denote the *flexible aggregate distance* for simplicity in the following. Our goal is to retrieve a point p^* in P to minimize d^p. An FANN$_{\mathcal{R}}$ query can be formalized with the following definition:

Definition 2 (FANN$_{\mathcal{R}}$ query)**.** The input of an FANN$_{\mathcal{R}}$ query is a quintuple (G, P, Q, ϕ, g), which returns a triple (p^*, Q^*_ϕ, d^*) as its answer such that:

$$\begin{cases} (p^*, Q^*_\phi) = \underset{p \in P, Q_\phi \subset Q, |Q_\phi| = \phi|Q|}{\operatorname{argmin}} g(p, Q_\phi), \\ d^* = g(p^*, Q^*_\phi), \end{cases}$$

where p^* is the point in P that minimizes the flexible aggregate distance, Q^*_ϕ is the optimal flexible subset, and d^* is the flexible aggregate distance.

B. Related Works

The shortest path algorithm. The shortest path algorithm is one of the most fundamental operations in road networks, and it has been extensively studied during the past half century. The *Dijkstra* algorithm [12] and its variants (e.g., A^* algorithm [13]) have been widely applied in location-based services. We can use either lower bounds (or other heuristic properties) or materialization techniques to accelerate the shortest path computation. The fully materialization of distances requires high storage cost, while *HiTi* [14] and *HEPV* [15] materialize distances partially to make it feasible for large graph. Currently, PHL [16] is the fastest method by decomposing a graph into the shortest paths and storing distances from each vertex to the shortest path in its labels. Note that many indexing techniques also rely on distance materializations, and we will discuss it in the following.

Indexing techniques and hierarchical structures. Indexing techniques and hierarchical structures [14], [15], [17]–[19] are also widely used in road networks. The basic idea is to partition the graph into subgraphs recursively, and pre-compute some shortcuts within subgraphs. It is usually required to keep the hierarchical structure balanced for better performance. *CH* [18] has a low memory overhead, but it has to traverse a large number of nodes when objects are relatively dispersed in the graph. The authors in [7], [20] transplanted Voronoi digram to the domain of road networks, but it often causes unbalanced partitions. K. C. Lee et al. [19] used *ROAD* to index road networks in a hierarchical way, but it performs badly if the objects are sparse and road networks are large. G-tree [11], [21] has a superior performance while the cost of building index is acceptable.

kNN, ANN, and FANN in the Euclidean space. The k-nearest neighbor (kNN) query has been studied for decades. Many successful approaches have been developed to solve this problem [11], [22], [23]. Abeywickrama et al. [24] studied the different in-memory algorithms for kNN queries, and they proved that IER has an excellent potential. The ANN problem has been studied in both the Euclidean space [1]–[3] and road networks [4]–[7], and successfully adapted to the top-k algorithms. The *IER* algorithm in [4] can reach the best performance, which uses R-tree to index the data objects, but it is not efficient when only partial points in Q are considered. The algorithms in [6], [7] use a Voronoi diagram to partition the road network, but they can often cause unbalanced partitioning and thus are also inefficient in large road networks. Y. Li et al. [8], [9] first studied FANN in the Euclidean space, which generalized the classical ANN problem

and offered it richer semantics. They proposed a series of exact and approximate algorithms to address this problem, but those algorithms cannot be used directly due to the complexity in road networks.

C. Outlines of Proposed Methods

In this paper, we design two kinds of algorithms which are denoted as the universal and specific methods respectively. The universal methods are able to deal with *any* g (i.e., suitable for both max and sum), and the specific methods can only solve the problem when g is either max or sum.

As for the universal methods, a naive way is to regard the ANN as an independent module. To be specific, we enumerate $\binom{|Q|}{\phi|Q|}$ options to determine Q_ϕ^p (as the query objects), and then apply the ANN routine (e.g., *IER* in [4]) directly. However, this method is always infeasible in practice since $\binom{|Q|}{\phi|Q|}$ is often too large to deal with. For example, the $\binom{|Q|}{\phi|Q|}$ can reach 2.39×10^{37} if we set $|Q|$ and ϕ to 128 and 0.5 respectively. To this end, we design a Dijkstra-based algorithm to compute g_ϕ (shown in Section III-A). Although it also searches in an enumerative way, it is much more efficient than the naive solution. Inspired by the threshold algorithm [25], *List* [8], [9] is proposed to answer FANN queries in the Euclidean space. We modify this *List*, and implement the modified algorithm in a "switchable" way to answer FANN$_\mathcal{R}$ queries in road networks. We denote it as *R-List* (shown in Section III-B). The modified implementation also leads to much more efficient solution to *max*-FANN$_\mathcal{R}$ problems (shown in Section IV-A). As presented in [24], IER has an excellent potential when retrieving kNN. Hence, we further design a IER-kNN framework to answer FANN$_\mathcal{R}$ queries. Based on the general IER-kNN framework, we can have a family of algorithms (shown in Section III-C).

Although the universal methods can generally achieve good performance, they highly rely on sophisticated indexing techniques. The construction cost of index can often be very high especially for frequently changing road networks. Motivate by this, we design a specific algorithm when g is max, which can return an exact answer, and denote it as *Exact-max* (shown in Section IV-A). *Exact-max* follows the basic data structure of *R-List*, but it often outperforms any other methods when it is index-free. What is more, it is often desirable to obtain a near-optimal result. To this end, we design a *APX-sum* algorithm to answer *sum*-FANN$_\mathcal{R}$ queries in road networks, which can return a 3-approximation result. We further prove that it can even return a 2-approximation result if Q is the subset of P (shown in Section IV-B).

III. UNIVERSAL METHODS

In this section, we present the universal solutions to answer both *sum*-FANN$_\mathcal{R}$ and *max*-FANN$_\mathcal{R}$ queries.

A. The Dijkstra-based Algorithm

The Dijkstra-based algorithm is based on the following observation. Recall how *Dijkstra* routine runs: at every step of its expansion, it chooses an unvisited nearest node of the source node to visit and updates its neighbors' distances to the source node. This behavior also makes sense in running $g_\phi(p, Q)$. First, let p be the source node, we call a *Dijkstra*-like routine on it. Then we keep the path expanding until $\phi|Q|$ nodes in Q are labeled as visited. Hence, these $\phi|Q|$ nodes are exactly Q_ϕ^p. Thus, we can enumerate points in P and return the one with the smallest flexible aggregate distance.

At a high level, the difference between the naive method mentioned in Section II-C and the Dijkstra-based algorithm is that: the former is to construct a Q_ϕ first, and then to determine the correct p and its flexible aggregate distance; the latter is to choose a p first, and then retrieve the correct Q_ϕ^p and its flexible aggregate distance.

Algorithm complexity. Since g_ϕ has the same time cost with *Dijkstra*, i.e., $O(|E| + |V| \log |V|)$ (assume that the min-priority queue is implemented by a Fibonacci heap), the total time cost is $O((|E| + |V| \log |V|)|P|)$. In the worst case, the space cost is $O(|P| + |Q| + |V| + 2\phi|Q|) = O(|V|)$.

Intuitively, there are two ways to improve this algorithm: prune nodes in P as much as possible (i.e., reduce the invocation of g_ϕ), or improve the implementation of g_ϕ. We will discuss related improving techniques in the following. For example, *R-List* (Section III-B) aims to reduce the invocation of g_ϕ, and IER-kNN (Section III-C) combines the two ways.

B. The R-List Algorithm

The basic idea of *R-List* algorithm is to construct a threshold [25] for early terminating, and thus it is able to reduce the invocation of g_ϕ. The *R-List* (road networks' *List* [8], [9]) is actually queue-based. We create $|Q|$ queues. Each queue corresponds to an object in Q, and processes data points in a from-near-to-far way.

Although *R-List* shares the basic idea with *List* [8], [9], we have two contributions here. Firstly, the implementation details of constructing the *list* of queues are different in the context of road networks. Secondly and most importantly, the modified implementation can lead to a much more efficient solution to *max*-FANN$_\mathcal{R}$ problems. To this end, we will discuss the detailed implementation and its algorithm complexity together with *Exact-max* in Section IV-A.

C. IER-kNN Framework

In this section, we propose a powerful algorithm framework for FANN$_\mathcal{R}$ query. Firstly, we adopt the Incremental Euclidean Restriction (IER) to prune the unnecessary nodes in P as much as possible. Let e be an entry of an R-tree that indexes P and b its *minimum bounding rectangle* (MBR). We can calculate the minimum possible distance from b to a point q, denoted as $mdist(b, q)$. Generally, let b' be another MBR, and we also use $mdist(b, b')$ to denote the minimum possible distance from b to b'. We denote g^ϵ as the *Euclidean aggregate function*, and we have

$$g^\epsilon(p, Q) = g\{\delta^\epsilon(p, q_1), \delta^\epsilon(p, q_2), \ldots, \delta^\epsilon(p, q_{|Q|})\}.$$

Similarly, $g^\epsilon(e, Q)$ can be defined by $g\{mdist(b, q_i), \forall q_i \in Q\}$. Like Definition 1, we can define the *flexible Euclidean aggregate function* (Euclidean FANN) by $g_\phi^\epsilon(p, Q)$ if replacing

$g(p, Q_\phi)$ with $g^\epsilon(p, Q_\phi)$. Following the similar definition, we also have $g^\epsilon_\phi(e, Q)$ to denote the flexible Euclidean aggregate function with respect to an MBR e and Q. For simplicity, we also use g^ϵ_ϕ to denote its flexible Euclidean aggregate distance if the context is clear. Now, we can prove the following lemma:

Lemma 1. Let Q be a set of query points and e an R-tree node entry. For any point p indexed under e, $g^\epsilon_\phi(e, Q)$ cannot be larger than $g_\phi(p, Q)$.

Proof: It is true due to $g^\epsilon_\phi(e, Q) \leq g^\epsilon_\phi(p, Q)$ and $g^\epsilon_\phi(p, Q) \leq g_\phi(p, Q)$. ■

Based on Lemma 1, we can solve the FANN$_\mathcal{R}$ query using an R-tree built on P. We show this process in Algorithm 1. Suppose *head* is a function get the head element from a queue. Initially, the root of the R-tree is enqueued into a priority queue which is sorted by $g^\epsilon_\phi(e, Q)$ in *ascending* order (line 2). For each iteration, we first check whether $g^\epsilon_\phi(e, Q)$ is larger than or equal to the current best candidate result (line 5). If so, we terminate the algorithm (line 6); otherwise, we check whether the dequeued item is an R-tree node (line 8). If so, we push all entries under this node into the priority queue (line 9-10); otherwise, we run $g_\phi(e, Q)$ on it and update the result if necessary (line 12-14). Note that in line 9, the entry $\hat{e}$ is a data point in P if e is a leaf node, and it is an R-tree node if e is a non-leaf node.

Algorithm 1: IER-kNN Framework

```
   Input: G, P, Q, φ, g, R
   Output: p*, Q*_φ, d*
1  d* ← ∞, H ← new priority queue
2  H.enqueue(R.root, g^ε_φ(R.root, Q))
3  while H is not empty do
4      e ← H.top()
5      if g^ε_φ(e, Q) ≥ d* then
6          break
7      H.dequeue()
8      if e is an R-Tree node then
9          foreach R-Tree entry ê under e do
10             H.enqueue(ê, g^ε_φ(ê, Q))
11     else
12         (Q^e_φ, d^e) ← g_φ(e, Q)
13         if d^e < d* then
14             p* ← e, d* ← d^e, Q*_φ ← Q^e_φ
```

A running example. Let us see how Algorithm 1 finds the sum-FANN$_\mathcal{R}$ in Fig. 1 whose $\phi = 50\%$. We illustrate this process in Fig. 2, and remove the road segments for better visualization. In the first round of loop, we would push $(MBR_1, 8), (MBR_2, 1.5)$, and $(MBR_3, 26)$ into H. After that, $(MBR_2, 1.5)$ will be popped, and we would check p_3 and p_6 within MBR_2. Clearly, d^* will be 4. We can safely terminate the algorithm since d^* is less than the distance value of head in H.

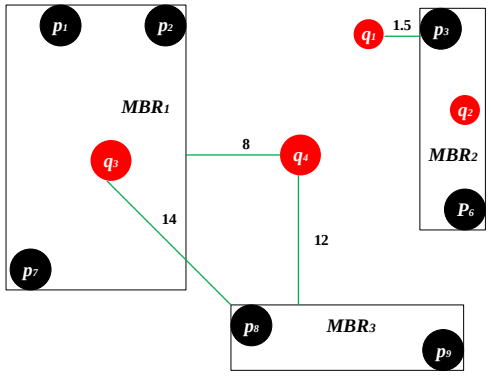


Fig. 2. Example of Algorithm 1

Revisitation of $g_\phi(p, Q)$. Now we revisit the implementation of $g_\phi(p, Q)$. As implied in Section III-A, given a p, Q and ϕ, $g_\phi(p, Q)$ is exactly an *incremental network expansion* (INE), which is also a kNN query where p is a query node and Q is the set of data objects and $k = \phi|Q|$.

Note that [4] uses A* to compute the shortest path distance due to its better performance compared to Dijkstra. However, A* is necessarily no better than INE when it comes to the implementation of g_ϕ, as showed in our experiments. This finding is also noted by [24]. To boost efficiency of computing the shortest path distance and kNN, we apply two state-of-the-art techniques. The first is G-tree [11], [21], which materializes distance matrix for each tree node. The second is *pruned highway labelling* (PHL) [16], which accelerates the shortest path distance queries by decomposing a graph into shortest paths and storing distances from each vertex to the shortest path in its labels. We denote the IER-kNN framework combined with the INE algorithm as IER-INE. Similarly, we also have IER-A*, IER-GTree and IER-PHL to represent the IER-kNN framework combined with A*, G-tree and PHL respectively.

$g_\phi(p, Q)$ itself can also be implemented with IER and any shortest path distance algorithm when Q is indexed under R-tree. In this way, we denote the IER-kNN method whose g_ϕ is implemented with IER-A* as IER^2-A* (there are double IER routines). The IER-A* here means the kNN (or g_ϕ) method, instead of the FANN$_\mathcal{R}$ approach mentioned above. Similarly, if we replace A* here with GTree (or PHL), we also have IER^2-GTree (or IER^2-PHL). Note that the "GTree" in IER-GTree is the kNN algorithm presented in [11], [21], which is based on an *occurrence list* (Occ) over Q, and the "GTree" in IER^2-GTree denotes the shortest path distance algorithm based on G-Tree index.

In this way, we can have a family of algorithms based on IER-kNN. The index techniques of different implementations for g_ϕ are summarized in Table I, and they will be further studied in the experimental section.

Note that we can also boost the efficiency of any proposed method by leveraging these index structures. We will exploit

TABLE I
ROAD NETWORK INDEX OF g_ϕ

Algorithm Name	G-tree	PHL	R-tree	Occ
INE	✗	✗	✗	✗
A*	✗	✗	✗	✗
GTree	✓	✗	✗	✓
PHL	✗	✓	✗	✗
IER-A*	✗	✗	✓	✗
IER-GTree	✓	✗	✓	✗
IER-PHL	✗	✓	✓	✗

it in Section VI.

Algorithm complexity. In the worst case, we still need to visit each point in P. Take the IER-GTree for an example, and the worst time cost of kNN search [11], [21] is $O(|V|\log|V|)$. The total time cost is $O(|P||V|\log|V|)$. As noted in [11], [21], the complexity is much smaller than the worst case complexity in practice. Because the space cost of R-tree or Occ is negligible compared with G-tree, the space cost is mainly made up of the cost of G-tree [11], [21], i.e., $O(|V|+|V|\log|V|+|E|)$.

In addition, we can also use another bound in line 5 of Algorithm 1 when we apply IER^2-A*, IER^2-GTree, or IER^2-PHL, which is not as tight as $g^\epsilon_\phi(e,Q)$, but can be computed cheaply. The new bound $d(p,Q)$ can be defined by:

- $d(p,Q) = mdist(b_Q,p)$ when g is max.
- $d(p,Q) = \phi|Q| \times mdist(b_Q,p)$ when g is sum.

where b_Q is the MBR of Q, and p can be either a data point or an R-tree node. It is obvious that we can terminate the algorithm if $d(e,Q) \geq d^*$ holds.

IV. SPECIFIC METHODS

Although the universal algorithms for FANN$_\mathcal{R}$ queries in Section III can achieve excellent performance, they highly depend on the sophisticated index techniques (e.g., G-tree or PHL) as shown in our experimental results. In this section, we design two specific algorithms to solve *sum*-FANN$_\mathcal{R}$ and *max*-FANN$_\mathcal{R}$ queries respectively. Most importantly, these specific methods can achieve excellent performance even if the underlying road networks are index-free.

A. The Exact-max Algorithm

We present this method in Algorithm 2, and call it *Exact-max* (exact *max*-FANN$_\mathcal{R}$), which shares the similar idea and data structure of *R-List*. The main difference is that we add a counter for every point in P. Initially, these counters are set to 0 (line 2). During every iteration, we get the head node with the smallest distance (line 4), and then increase the counter associated with the head node by one (line 5). If the counter associated with the head node reaches $\phi|Q|$, the head node is exactly p^*, and then we can terminate the algorithm safely (line 6-9). Hence, we can run the time consuming g_ϕ only once (line 8). This is why *Exact-max* can be efficient. Besides, this also indicates that **the different implementations of g_ϕ have little influence on *Exact-max***. In other words, we may get an adorable performance even if we do not build a road network index over the whole G. This property is appealing when road networks change frequently, since we do not need to re-build the index any more, which is usually time consuming as shown in Fig. 9(b).

Algorithm 2: The *Exact-max* Algorithm

Input: G, P, Q, ϕ, δ, g
Output: p^*, Q^*_ϕ, d^*

1 **foreach** $p \in P$ **do**
2 $count[p] \leftarrow 0$
3 **while** *true* **do**
4 $L_{min} \leftarrow$ the queue whose head has the smallest distance
5 $count[L_{min}.top()] \leftarrow count[L_{min}.top()] + 1$
6 **if** $count[L_{min}.top()] \geq \phi|Q|$ **then**
7 $p^* \leftarrow L_{min}.top()$
8 $(Q^*_\phi, d^*) \leftarrow g_\phi(L_{min}.top(), Q)$
9 break
10 $L_{min}.dequeue()$

Implementation details. Now we discuss the implementation details of *R-List* and *Exact-max*. The *list* of queues is constructed in an implicit way, or it will violate the memory limit if $O(|P||Q|)$ is larger enough. To be specific, We set the $|Q|$ query nodes instead of p as the multiple sources initially, and then execute *Dijkstra*-like routine on them simultaneously and independently. As shown in Algorithm 2, the queue operations are alternately performed on different queues, which implies that we can implement this *multi-source Dijkstra* procedure in a "switchable" way. To be specific, all data structures related to different queues should be well preserved when the *Dijkstra*-like routine switches away, thus the interrupted search process can be reloaded and resumed when it switches back.

A running example. Let us see how *Exact-max* finds the max-FANN$_\mathcal{R}$ in Fig. 1 whose $\phi = 50\%$. The expanding paths of $\{q_1,q_2,q_3,q_4\}$ are $\{p_3,\dots\}$, $\{p_3,\dots\}$, $\{p_4,\dots\}$ and $\{p_5,\dots\}$ respectively. It is obvious that the counter of p_3 will reach $\phi \times 4 = 2$ first. Hence the result of this max-FANN$_\mathcal{R}$ query is $p^* = p_3$, $d^* = 2$ and $Q^*_\phi = \{q_1,q_2\}$.

The correctness of this algorithm is easy to validate: the nature of the *Dijkstra* algorithm guarantees that the closer nodes to the source are, the earlier they will be visited. When a node is the first to be visited by exact $\phi|Q|$ sources, it means that this node is the closest one to the $\phi|Q|$ sources. Thus Algorithm 2 can answer *max*-FANN$_\mathcal{R}$ queries correctly.

Note that basic idea of *Exact-max* is different from g_ϕ which is implemented with *Dijkstra* or *INE*. The latter is to regard P as sources and then obtain the kNN (i.e., the nodes in Q are destinations). The expansion direction of *Exact-max* is quite the reverse: we regard the nodes in Q as sources and then expand them to P (i.e., the nodes in P are destinations). It is not hard to understand that *Exact-max* is very efficient when

P is dense and $\phi|Q|$ is relatively small. In most reality cases, this is true because $|Q|$ is much smaller than $|P|$.

Algorithm complexity. Assume that g_ϕ is implemented by *Dijkstra*-like method. For *R-List* algorithm, every point in P will be visited in the worst case. Thus, the time cost is $O((|E| + |V|\log|V|)|P|)$. In practice, the time complexity is often smaller than it due to the lower bound. The space cost is $O(|Q||V|)$ in the worst case, and it is mainly made up of the *list* of queues. Similarly, the time cost of *Exact-max* is $O(|E| + |V|\log|V|)$, and its space cost also includes the counter usage, i.e., $O(|Q||V|+|P|) = O(|Q||V|)$ in the worst case.

TABLE II
A COUNTER EXAMPLE OF SUM-FANN$_\mathcal{R}$

Source	Expanding		
q_1	(4, p_2)	(12, p_3)	-
q_2	(2, p_1)	(10, p_2)	-
q_3	(11, p_1)	-	-
q_4	(14, p_4)	-	-
q_5	(15, p_2)	-	-

It is worth noting that this method cannot be used to answer *sum*-FANN$_\mathcal{R}$ queries. Table II illustrates a counter example. Suppose query nodes set Q is $\{q_1, q_2, q_3, q_4, q_5\}$ (any query node does not belong to $\{p_1, p_2, p_3, p_4, p_5\}$), and $\phi = 40\%$. Hence, $\phi|Q| = 2$. If we follow the idea of Algorithm 2, we would examine the queue of q_2 first, and visit p_1. Secondly, we would examine the queue of q_1, and visit p_2. Thirdly, we would examine the queue of q_2 again, and hence visit p_2 again. At this moment, the counter of p_2 reaches 2, and we can terminate the algorithm. In this way, we have $p^* = p_2$, $d^* = 4+10 = 14$, and $Q^*_\phi = \{q_1, q_2\}$. However, the correct answer is $p^* = p_1$, $d^* = 2 + 11 = 13$, and $Q^*_\phi = \{q_2, q_3\}$.

B. The APX-sum Algorithm

For the *sum*-FANN$_\mathcal{R}$ problem in road networks, we present an approximate approach *APX-sum* (approximate *sum*-FANN$_\mathcal{R}$) in Algorithm 3. This algorithm is extremely simple, but it has a constant approximation ratio. Instead of considering the whole P, we only examine those data points which are the nearest neighbors of those query nodes in Q (line 2-4). Then we regard the candidate set as P, and run the FANN$_\mathcal{R}$ algorithm (whose g is sum). Hence, we reduce the number of candidate data points to $|Q|$, which is usually much smaller than $|P|$. This is why it can remarkably improve the search efficiency. Actually, it is even possible that the size of candidate set is smaller than $|Q|$, since different query points may have the same nearest data point neighbor. One of the most appealing properties of *APX-sum* is the stability when varying P because it is only affected by Q generally. We can prove that the approximation ratio d^α/d^* of this algorithm is no more than 3.

Theorem 1. Algorithm 3 returns a 3-approximation answer to any *sum*-FANN$_\mathcal{R}$ query in road networks.

Algorithm 3: The *APX-sum* Algorithm

Input: G, P, Q, ϕ, δ, g
Output: p^α, Q^α_ϕ, d^α

```
1 candidate ← ∅
2 foreach q ∈ Q do
3     p ← the nearest neighbor of q in P
4     candidate.insert(p)
5 FANN_R (G, candidate, Q, φ, sum)
```

Proof: Given an *sum*-FANN$_\mathcal{R}$ query, Algorithm 3 returns an approximate answer $(p^\alpha, Q^\alpha_\phi, d^\alpha)$, and the true optimal answer is (p^*, Q^*_ϕ, d^*). Let q^τ be the nearest node in Q^*_ϕ to p^*, and p^τ be the nearest node in P to q^τ. We have:

$$\delta(p^\tau, q^\tau) \le \delta(p^*, q^\tau) \tag{1}$$

And for any $q \in Q^*_\phi$, we have:

$$\delta(p^*, q^\tau) \le \delta(p^*, q) \tag{2}$$

$$\Rightarrow \phi M \cdot \delta(p^*, q^\tau) \le \sum_{q \in Q^*_\phi} \delta(p^*, q) = d^* \tag{3}$$

It is obvious that p^τ cannot be "better" than p^α. If the result of $g_\phi(p^\tau, Q)$ is (Q^τ_ϕ, d^τ), we have:

$$\begin{aligned}
d^\alpha &\le d^\tau \\
&= \sum_{q \in Q^\tau_\phi} \delta(p^\tau, q) \\
&\le \sum_{q \in Q^*_\phi} \delta(p^\tau, q) \\
&\le \sum_{q \in Q^*_\phi} (\delta(p^\tau, p^*) + \delta(p^*, q)) \qquad \text{(triangle inequality)} \\
&= \sum_{q \in Q^*_\phi} \delta(p^\tau, p^*) + d^* \\
&= \phi M \cdot \delta(p^\tau, p^*) + d^* \\
&\le \phi M \cdot (\delta(p^\tau, q^\tau) + \delta(q^\tau, p^*)) + d^* \\
&\le 2\phi M \cdot \delta(q^\tau, p^*) + d^* \qquad \text{(by Eqation 1)} \\
&\le 2d^* + d^* \qquad \text{(by Eqation 3)} \\
&= 3d^*
\end{aligned}$$

■

A running example. Let us see how *APX-sum* finds the sum-FANN$_\mathcal{R}$ in Fig. 1 whose $\phi = 50\%$. First, we can easily obtain the candidates of data points $\{p_3, p_4, p_5\}$. Since the true optimal p^* belongs to the set of candidates, *A-sum* returns $p^* = p_3$, $d^* = 4$ and $Q^*_\phi = \{q_1, q_2\}$ as the final result.

Algorithm complexity. *APX-sum* is consisted of finding the nearest neighbors and FANN$_\mathcal{R}$. If the g_ϕ is implemented as *Dijkstra* or INE, the time cost is $O((|E| + |V|\log|V|)|Q|)$. The space cost is $O(|V|)$ in the worst case.

It should be pointed out that, 3 is only a theoretical bound. As shown in our experiments, the approximation ratio of *APX-sum* is far less than 3 in practice: it never exceeds 1.2 in our experiments. We further propose a new theorem when Q is the subset of P in the following.

Theorem 2. Algorithm 3 returns a 2-approximation answer to any *sum*-FANN$_{\mathcal{R}}$ query in road networks if Q is a subset of P.

Proof: If Q is the subset of P, p^τ is also q^τ in the proof of Theorem 1, and hence $\delta(p^\tau, q^\tau) = 0$ holds. Thus, $d^\alpha \leq 2d^*$ is true if we replace $\delta(p^\tau, q^\tau)$ with 0 in the proof of Theorem 1. ■

V. Extension To The k-FANN$_{\mathcal{R}}$ Problem

In this section, we define the k-FANN$_{\mathcal{R}}$ query, which is a further extension of FANN$_{\mathcal{R}}$. We can regard FANN$_{\mathcal{R}}$ as a special case of k-FANN$_{\mathcal{R}}$ when k is 1.

Definition 3 (k-FANN$_{\mathcal{R}}$ query)**.** A k-FANN$_{\mathcal{R}}$ query is a six-tuple (G, P, Q, ϕ, g, k), which returns a k-element vector X as its answer. Each element of X is a pair (p_i, r_i), in which $i = 1, 2, \cdots, k$, $p_i \in P$ and r_i is the flexible aggregate distance from p_i to Q. For any node $p_0 \in P \backslash \{p_1, p_2, \cdots, p_k\}$, assume its flexible aggregate distance to Q is r_0, we have $r_0 \geq \max\{r_1, r_2, \cdots, r_k\}$.

Obviously, it is not necessary for $Q_\phi^{p_i}$ to be the same, where $i = 1, 2, \cdots, k$. With some minor modifications, all of the algorithms in this paper can be easily adapted to answer the k-FANN$_{\mathcal{R}}$ query except the *APX-sum* algorithm.

The Dijkstra-based algorithm. To get the top-k, we need to update the queue when enumerating the P. Finally, the queue is our final result.

The *R-List* algorithm. The threshold τ is calculated in the same way as the original *R-List* algorithm. Instead of comparing τ with the smallest distance, we compare it with the k-th smallest distance in the priority queue. If τ is larger, we can terminate the algorithm and return the priority queue as the k-FANN$_{\mathcal{R}}$ answer.

The IER-*k*NN framework. Like the adaptation for the *R-List* algorithm, instead of comparing $g_\phi^\epsilon(e, Q)$ with the smallest distance, we compare it with the k-th smallest distance in the priority queue. If $g_\phi^\epsilon(e, Q)$ is larger, we can terminate the algorithm and return the priority queue as the k-FANN$_{\mathcal{R}}$ answer.

The *Exact-max* algorithm. For the k-FANN$_{\mathcal{R}}$ problem, we should expand the paths until k different counters reach $\phi|Q|$. Then, we can terminate the search routine and return the k corresponding query nodes and their flexible distances as the final answer to the k-FANN$_{\mathcal{R}}$ query.

TABLE III
ROAD NETWORK DATASETS

Name	Description	# nodes	# edges
DE	Delaware	48,812	119,004
ME	Maine	187,315	412,352
COL	Colorado	435,666	1042,400
NW	Northwest USA	1,089,933	2,545,844
E	Eastern USA	3,598,623	8,708,058
CTR	Central USA	14,081,816	33,866,826
USA	Full USA	23,947,347	57,708,624

VI. Experiments

A. Setup

We implemented all the algorithms mentioned in this paper by standard C++, and executed our experiments on a Linux machine with dual 6-core Intel Xeon E5-2620 processors at 2.00 GHz and 64 GB DDR3 RAM. All of our road network datasets come from the real word[1]. Note that the original datasets have many errors, such as unconnected components or self-loops, and we have cleaned it up at the preprocessing stage. We show them in Table III.

Given a road network, there are many factors that will affect the cost of an FANN$_{\mathcal{R}}$ query. In our experiments, we mainly focus on those in the following:

- d, the density of P
- A, the coverage ratio of Q
- M, the size of Q (i.e., $|Q|$)
- C, the number of clusters of Q
- ϕ, the flexibility parameter

where d controls the generation of P, and A, M and C affect the generation of Q.

Uniform data points. Parameter d reflects the ratio of the size of P to the size of nodes in whole graph (i.e., $|P|/|V|$). We assume that P is generated randomly in the road network given a density.

Uniform query points. Parameter A reflects aggregation degree of the query nodes in Q. We first randomly select a node in V as a source node (i.e., *seed* node), and calculate the shortest path distances from it to all other nodes in V. We denote the maximum one as the *radius* of G. Next, we randomly choose M nodes from G, whose distances to the seed node are no more than $A \times radius$. Note that we assume the size of nodes in such region is always larger than or equal to M. If there is no enough objects in the region, we simply expand outward until the size reaches M.

Clustered query points. In some scenarios, the query points are not uniformly distributed. Some locations, such as schools, often occur in clusters. After we determine a region by A in the road network, we select C central nodes in the selected region, and choose M/C nodes in the vicinity of each central node by expanding from it.

Real-world POIs. We use the real-word data from [24], which

[1] http://www.dis.uniroma1.it/challenge9/download.shtml

is extracted from OpenStreetMap (OSM)[2]. We only show the POIs within NW since our default road network is NW (see Table IV).

TABLE IV
REAL WORLD POIS IN NW

Name	Description	# nodes	Density
PA	Parks	5,098	0.005
SC	Schools	4,441	0.004
FF	Fast Food	1,328	0.001
PO	Post Offices	1,403	0.001
HOT	Hotels	460	0.0004
HOS	Hospitals	258	0.0002
UNI	Universities	95	0.00009
CH	Courthouses	49	0.00005

We use the NW as our default road network. We vary d from 0.0001 to 1, and vary A from 1% to 20%, and vary M from 64 to 1024, and vary C from 1 to 8, and vary ϕ from 0.1 to 1. The default values of d, A, M, C and ϕ are set to 0.001, 10%, 128, 1 and 0.5 respectively. These values are set stochastically and others are also adoptable. By default, both P and Q are generated uniformly. In order to minimize the randomness, we average the results of algorithms over 100 queries. The performances of R-tree and G-tree are dependent on the value of fanout f (both R-tree and G-tree) and maximum number of points in a leaf node τ (G-tree). In our experiments, we set f to 4. And for G-tree, we set τ to 64 (DE), 128 (ME, COL), 256 (NW, E), and 512 (CTR, USA) respectively.

In the following paper, we use *GD* to denote the family of generalized Dijkstra-base algorithms, since the *Dijkstra* here can be regarded as INE and it can be replaced with other g_ϕ methods (e.g., A*, PHL, and G-tree). In the meanwhile, the specific Dijkstra-based algorithm (see Section III-A) is denoted as *Baseline*. Note that we only show the results of *max*-FANN$_\mathcal{R}$ for universal methods (i.e., *GD*, *R-List*, and IER-kNN) due to the space limit except the evaluation for approximation ratio in Section VI-C. Fortunately, the running time of *sum*-FANN$_\mathcal{R}$ is very close to that of *max*-FANN$_\mathcal{R}$ given the same input, which is showed in Appendix C of our full paper [26] due to the space limit.

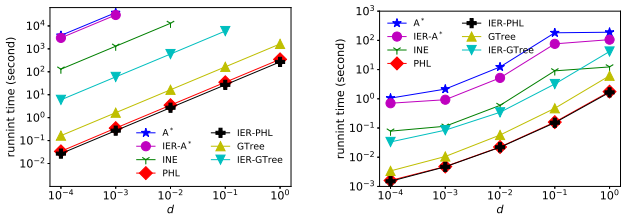
B. Evaluation for Efficiency

The evaluation of efficiency includes the query efficiency of different algorithms, the index cost, and the extended k-FANN$_\mathcal{R}$ queries.

1) Query Efficiency: The experimental results of different algorithms' running time (excluding the construction time of index) are shown in the following, with respect to the varying parameters of d, A, M, C and ϕ.

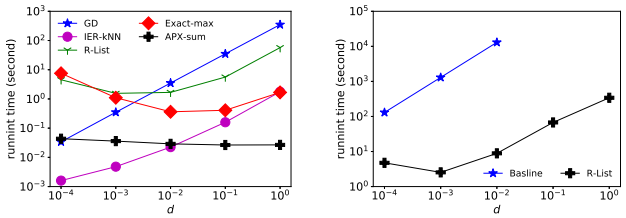
Varying d. We use $0.0001, 0.001, 0.01, 0.1, 1$ to vary d. Firstly, we present the experimental results of *GD* and IER-kNN algorithms which are implemented by different g_ϕ routines in Fig. 3. Note that the legends here refer to g_ϕ methods. We can find that PHL and IER-PHL always preform best while A* and IER-A* are worst among these different implementations. Another important conclusion is that there is a linear (or sub-linear) relationship between the running time and density of P for baseline (or IER-kNN) algorithms. The IER version of A* is slightly better than A* in generally. Comparing Fig. 3(a) with 3(b), we can see that IER-kNN outperforms *GD* by 1-3 orders of magnitude with respect to the same g_ϕ.

From Fig. 3(a), we can conclude that *GD* algorithm is often infeasible if it is combined with A*, IER-A*, or INE, which implies that *Baseline* is often infeasible. For example, we can observe the gap between *Baseline* and *R-List* (whose g_ϕ are both implemented with INE) with respect to the running time (see Fig. 4(b)), and the *Baseline* cannot finish the query when d is larger than 10^2 within a reasonable time. We will discuss more about this based on the results of Fig. 4(b) in Section VI-E. And since PHL (or IER-PHL) is the most efficient implementation of g_ϕ, we choose PHL as the default implementation of g_ϕ in the latter experiments.



(a) *GD* (b) IER-kNN

Fig. 3. Efficiency of *GD* and IER-kNN implemented by different g_ϕ



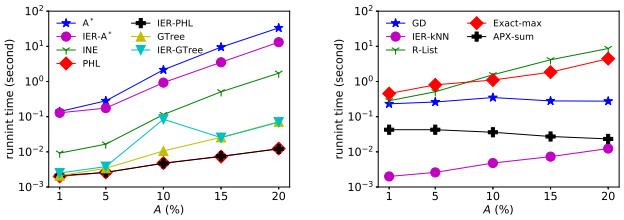
(a) All FANN$_\mathcal{R}$ algorithms (b) *R-List* and *Baseline*

Fig. 4. Efficiency when varying d

Fig. 4(a) shows the efficiency of all FANN$_\mathcal{R}$ algorithms presented in this paper. With the increasing of d, IER-PHL performs best at first, and *APX-sum* outperforms any other method when d is larger than 0.01. We also find that *R-List* is better than the *GD* algorithm when d is large. We can validate the stability of *APX-sum* when varying d, and this is because *APX-sum* depends much on Q instead of P. Both *APX-sum* and *Exact-max* have the tendency that they cost less time when d is larger. This is because the expanding routine from Q to P is faster when the data points is denser. The reason why *Exact-max* decreases first and then increases is due to the trade-off between the expanding overhead and convenience brought by d. To be specific, a larger d will lead to a higher expanding overhead in general, while it will also make the terminating condition to be fulfilled earlier.

[2]http://www.openstreetmap.org

Varying A. Now we study the efficiency with different coverage ratios of Q. We use $1\%, 5\%, 10\%, 15\%, 20\%$ to vary A. We show the results in Fig. 5.



(a) IER-kNN (b) All FANN$_{\mathcal{R}}$ algorithms

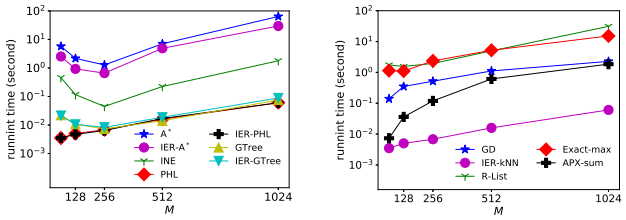
Fig. 5. Efficiency when varying A

We can conclude that IER-kNN can reflect how the efficiency is affected by different g_ϕ routines when varying A. Fig. 5(a) shows the results of IER-kNN implemented by different g_ϕ when we vary A. Clearly, PHL and IER-PHL outperform other methods. All algorithms will cost more running time with the increase of A. This is because larger A means that Q is sparser in road networks, and hence it will usually travel within a larger region. As shown in Fig. 5(a), the slopes of A*, IER-A* and INE are larger than those of others. This behavior is consistent with the their nature of "expanding".

Further, we show the results of all algorithms in Fig 5(b). Clearly, *APX-sum* is stable when we vary A. This validates our conclusion that *APX-sum* has little dependence on Q's sparsity. As for *R-List* and *Exact-max* algorithm, their performances are bad if Q is sparse, and this because sparer Q often leads to a slower expanding from Q to P. We can also find the *GD* algorithm is stable for different A. This is not surprising, as the *GD* method depends much on P. Finally, *R-List* and *Exact-max* are not better than *GD* algorithm here, and this is also verified by the efficiency results in Fig. 4(a) when $d = 0.001$.

Varying M. Our next experiment investigates the efficiency when varying the size of Q. We use $64, 128, 256, 512, 1024$ to vary M. The results are shown in Fig. 6. As for IER-kNN methods, they have the tendency that larger M leads to worse efficiency in general. Note that the running time decreases first (from $M = 64$ to $M = 256$) for most IER-kNN methods. This is due to the trade-off between M and the sparsity of Q. To be specific, the smaller M results in a larger sparsity given a coverage ratio of Q . We also find that IER version of A* can improve the performance generally. Besides, the differences among PHL, GTree, IER-PHL, and IER-GTree are minor in Fig. 6(a). From Fig. 6(b), we can clearly find that *APX-sum* increases with the increase of M, and this verifies our conclusion that *APX-sum* depends much on the size of Q.

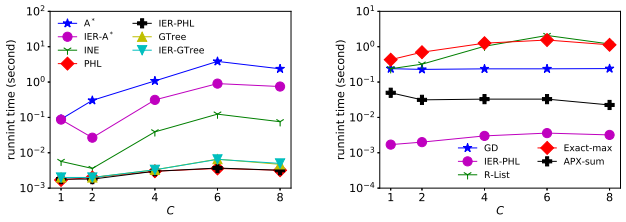
Varying C. We evaluate the effect of C if Q is generated in clusters, rather than uniformly. We use $1, 2, 4, 6, 8$ to vary C. Fig. 7 shows the results. Clearly, the larger C leads to worse performance in general, and this tendency is more serious for "expanding-based" methods in Fig. 7(a). When C is larger



(a) IER-kNN (b) All FANN$_{\mathcal{R}}$ algorithms

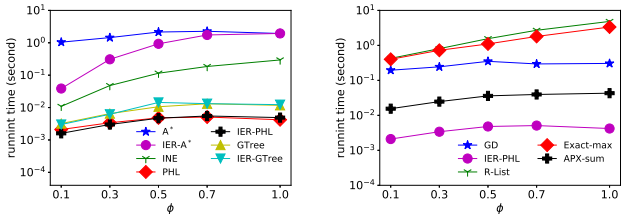
Fig. 6. Efficiency when varying M

enough, the performance will be stable and the running time will approximate to the value when Q is generated uniformly. For example, the running time of IER-A* is 2.16 seconds if the Q is generated uniformly, while the cost is 2.37 seconds when $C = 8$. As shown in Fig. 7(b), *R-List* and *Exact-max* are more affected by C due to their similar mechanisms.



(a) IER-kNN (b) All FANN$_{\mathcal{R}}$ algorithms

Fig. 7. Efficiency when varying C



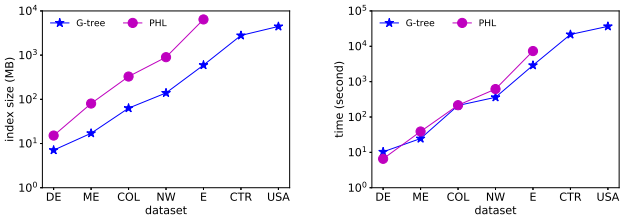
(a) IER-kNN (b) All FANN$_{\mathcal{R}}$ algorithms

Fig. 8. Efficiency when varying ϕ

Varying ϕ. The final part of query efficiencies is to study the effect of ϕ, i.e., the flexibility parameter. We use $0.1, 0.3, 0.5, 0.7, 1.0$ to vary ϕ. Fig. 8 shows our results. There is an obvious positive correlation with ϕ. This is reasonable because the larger ϕ means that more destinations need to be visited. We can find that R-tree over Q has less improvement for A* with the increasing of ϕ, however, the improvement is very obvious when ϕ is relatively small. We can also find that *R-List* and *Exact-max* are more effected by ϕ in Fig. 8(b).

2) Index Efficiency: Here we measure the construction time and size of the road networks' index structures used in our algorithms. The different index techniques have been presented in Table I.

Index cost of G-tree and PHL. Fig. 9 shows the index size



(a) Index size of G (b) Construction time of G

Fig. 9. Index cost of different road networks

and the construction time of G-tree and PHL for different datasets. This is also studied in [24]. Generally, G-tree costs less storage than PHL. Note that PHL only can build index for the first 5 datasets before exceeding the memory capacity. This experimental result suggests that G-tree is the only choice for very large road networks (e.g. USA), since PHL fails to build index for CTR and USA in a single commodity machine. It is obvious that the construction time of them is very close generally. We can conclude that the major considerations are the running efficiency and memory capacity rather than the index construction time when choosing between G-tree and PHL.

Index cost of R-tree and Occ. Due to the space limit, the experimental results are not showed here, and readers can refer to Appendix A in our full paper [26]. We can find that although Occ always has higher time and size cost than R-tree, the differences are quite trivial compared with the running cost. Hence, the index cost of Q can be ignored when we need to make a choice between GTree and IER-GTree.

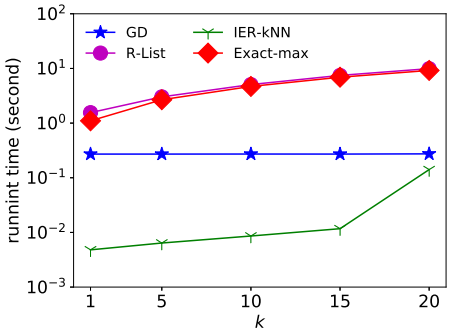


Fig. 10. Efficiency of k-FANN$_{\mathcal{R}}$

3) Efficiency of k-FANN$_{\mathcal{R}}$ Queries: Now we study performance of the k-FANN$_{\mathcal{R}}$ queries of different algorithms. We use $1, 5, 10, 15, 20$ to vary k. Fig. 10 shows the results. It is obvious that the query time will increase with the increasing of k except for the *GD* algorithm. The stability of *GD* is because that it mainly depends on P and the choice of g_ϕ, and other factors rarely have influence on it. Thus, the computation cost is similar with the case when $k = 1$ for *GD*. We can also notice that both the *Exact-max* and *R-List* algorithms are more sensitive to the increasing k than others. This is because they need more expanding overheads when k increases. **The reason why *GD* always comes in the second place will be discussed in Section VI-E.**

C. Approximation Quality of APX-sum

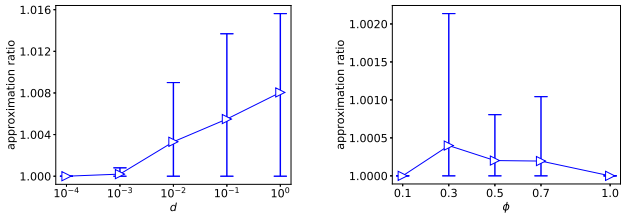
The *APX-sum* algorithm proposed in Section IV-B is an approximate algorithm. Due to the space limit, we only show the results of its approximation quality in Fig. 11 when varying d and ϕ. To get the results when varying other parameters, the readers can refer to Appendix B in our full paper [26]. The y-error in error bars is the standard deviation. The results show that *APX-sum* algorithm has an excellent approximation quality and is quite stable, and the approximation ratio is always less than 1.2 in all experimental cases.

D. Real World Data

We choose FF and PO POIs as our P since their densities are equal to our default density (i.e., 0.001) in Table IV. Note that there is no any type of POIs whose size is equal to our default size (i.e., 128), and hence we the POIs whose sizes are near to 128. In this way, we choose HOS and UNI POIs as our Q in Table IV.

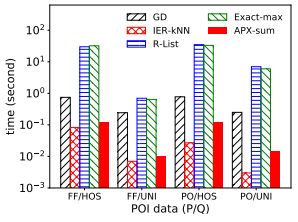
1) Query Efficiency: As shown in Fig. 12(a), the performance of different FANN$_{\mathcal{R}}$ methods on real world POIs has the similar characteristics with the evaluation using synthesized data. **Again, the reason why *GD* always comes in the third place will be discussed in Section VI-E.**

2) Approximation Quality of APX-sum: We call find that the *APX-sum* also has a very good approximation quality on real world POIs as showed in Fig. 12(b). The approximation ratio is always less than 1.1.



(a) Varying d (b) Varying ϕ

Fig. 11. Approximation quality of *APX-sum*



	Approximation ratio
FF/HOS	1.003
FF/UNI	1.0
PO/HOS	1.004
PO/UNI	1.076

(a) Effiency (b) Approximation quality

Fig. 12. Real world data

TABLE V
EFFICIENCY OF *Exact-max* WITH DIFFERENT g_ϕ (SECOND)

g_ϕ \ d	0.0001	0.001	0.01	0.1	1
A*	7.26	1.24	0.65	0.69	2.05
IER-A*	7.07	1.05	0.47	0.50	1.79
INE	6.81	0.94	0.35	0.38	1.62
PHL	7.56	1.10	0.36	0.41	1.67
IER-PHL	7.59	1.07	0.36	0.40	1.68
GTree	6.77	0.92	0.34	0.37	1.60
IER-GTree	6.78	0.93	0.34	0.38	1.64

E. Revisitation of GD Algorithm

As shown in experimental results above (Section VI-B1, VI-B3, VI-D1), we can find that the *GD* is even better than *R-List* (or *Exact-max*) in some scenarios if g_ϕ is implemented by PHL. There are two reasons: i) Our default d is set to 0.001 and the performance evaluation can be seen in Fig. 4(a); ii) **PHL is fast enough such that the advantage of *R-List* (or *Exact-max*) is shadowed**. We will investigate the second reason in the following.

To begin with, we show the experimental results of *Exact-max* implemented by different g_ϕ routines in Table V. Although we find that the efficiencies of different g_ϕ methods may differ sharply in Fig. 3, it has little influence on *Exact-max*. Clearly, *Exact-max* outperforms *GD* by 2 orders of magnitude if g_ϕ is implemented by A* even if $d = 0.0001$.

Next, we use $0.0001, 0.001, 0.01, 0.1, 1$ to vary d and compare the *R-List* with the *GD* algorithm when g_ϕ is implemented by INE. Fig. 4(b) shows the result. We can also conclude that *R-List* is much better than *GD* if there is no any precomputed index over the whole road network. There is not doubt that *Baseline* or *GD* is infeasible when road network index is not available (e.g., road networks in online games' maps).

VII. CONCLUSION

In this paper, we have studied an interesting problem of flexible aggregate nearest neighbor queries in road networks (FANN$_\mathcal{R}$). We propose a series of universal methods to solve this problem, including a Dijkstra-based algorithm, *R-List* and IER-kNN algorithm framework. Combined with the state-of-the-art shortest path techniques, the proposed algorithms can achieve excellent performance. Our specific approaches (*Exact-max* and *APX-sum*) sacrifice some generalities, but they are easier to implement and much more efficient than those universal approaches especially when the road networks are index-free. *Exact-max* can return an exact answer, and *APX-sum* can return a near-optimal result with a guaranteed 3-approximation. Finally, we successfully adapt most of the proposed algorithms to answer the extended k-FANN$_\mathcal{R}$ queries.

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